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RADIATIVE HEAT EXCHANGE IN COMBUSTION PROCESSES

NUMERICAL SOLUTION OF THE RADIATIVE-TRANSFER EQUATION FOR AN ABSORBING, EMITTING, AND SCATTERING MEDIUM WITH A COMPLEX 3-D GEOMETRY

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A method is proposed for numerically solving the integro-differential, radiative-transfer equation with the use of its piecewise-analytic solutions obtained by the discrete-ordinate method and grids constructed by the finite-element method. The advantages of the method proposed and some results of calculation of the radiative-transfer characteristics for one-, two-, and three-dimensional problems are discussed.

Radiative energy transfer is of crucial importance in many natural and technical processes of energy exchange. This pertains equally to high-temperature processes (combustion of organic fuels, thermal treatment of metals, high-temperature synthesis and pyrolysis in chemical technologies, etc.) where the radiative energy transfer accounts for 90% or more of the total energy exchange (see, for example, [1–4]) and to the processes occurring at lower atmospheric temperatures [5, 6]. It is known that an exact estimation of the characteristics of heat and mass transfer in technological processes allows one to obtain a significant economical effect, i.e., to increase the quality and functional characteristics of products and decrease their cost, as well as to make for good environmental conditions and to conserve material, energy, and manpower resources. For example, the temperature fields of many technological processes (several tens of them are described in [1, 2]) occurring at temperatures from 125 to 1600^{o}C should be calculated with an accuracy of ~1–2^{o}C.

Radiative heat exchange plays a dominant role in the total heat exchange in high-temperature processes in gaseous media. The accuracy of estimation of the temperature fields of such media is primarily dependent on the correctness of calculation of the radiative-transfer characteristics. This is also very important for optimization of the heating of steel products having a different geometry in ring furnaces with a moving bottom in which the working temperatures can reach 1200^{o}C.

Mathematical Model. It is difficult to calculate the characteristics of radiative heat transfer in selectively emitting, absorbing, and scattering media because, in this case, it is necessary to take into account the multiple processes of reradiation on solid particles, the selectivity of the radiation of gas components, and the temperature inhomogeneity and complex configuration of the radiating volume. The correctness of estimation of the radiative-heat-exchange characteristics depends, to a large extent, on the correctness of solution of the radiative-transfer equation [7–9]. In the case of a local thermodynamic equilibrium, this equation defines the law of conservation of radiant energy in the process of its propagation in an absorbing, emitting, and scattering medium:

$$\mathbf{l}\cdot\nabla I_\lambda(\mathbf{r},\mathbf{l}) + [\chi_\lambda(\mathbf{r}) + \sigma_\lambda(\mathbf{r})]\, I_\lambda(\mathbf{r},\mathbf{l}) = \chi_\lambda(\mathbf{r})\, B_\lambda(T(\mathbf{r})) + \frac{\sigma_\lambda(\mathbf{r})}{4\pi}\int_{4\pi} p_\lambda(\mathbf{r},\mathbf{l},\mathbf{l}')\, I_\lambda(\mathbf{r},\mathbf{l}')\, d\Omega' \,. \tag{1}$$

The boundary conditions for Eq. (1) are determined by the radiation and reflection processes occurring on the boundary surfaces of the medium and can be written, in the general case, as [9]

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$$I_\lambda(\mathrm{P},\mathbf{l})\big|_{(\mathbf{l}\cdot\mathbf{n})<0} = I_{0\lambda}(\mathrm{P},\mathbf{l}) + \frac{1}{\pi}\int_{2\pi} \rho_\lambda(\mathrm{P},\mathbf{l},\mathbf{l}')\, I_\lambda(\mathrm{P},\mathbf{l}')\,(\mathbf{l}'\cdot\mathbf{n})\, d\Omega' . \tag{2}$$

Having determined the radiation-intensity field from Eqs. (1) and (2), we may determine two more energy quantities necessary for calculating the temperature of the medium — the volume density of the radiation sources/heat flows at each point of the medium

$$\mathrm{div}\,\mathbf{Q}_\mathrm{r} = \int_0^\infty \chi_\lambda(\mathbf{r})\left(4\pi B_\lambda(T(\mathbf{r})) - \int_{4\pi} I_\lambda(\mathbf{r},\mathbf{l})\, d\Omega\right) d\lambda \tag{3}$$

and the local densities of the resulting radiant flux on the heat-absorbing surfaces (if they exist)

$$q_\mathrm{w}^\mathrm{r}(\mathrm{P}) = \int_0^\infty \varepsilon\left(\int_{2\pi} I_\lambda(\mathrm{P},\mathbf{l})\,(\mathbf{l}\cdot\mathbf{n})\, d\Omega - \pi B_\lambda(T_\mathrm{w}(\mathrm{P}))\right) d\lambda . \tag{4}$$

Examples of such surfaces include the lining of a furnace and the surfaces of steel products heated in this furnace.

Brief Review of Modern Methods of Solving the Radiative-Transfer Equation. At present there are a fairly large number of different methods of solving Eq. (1) with boundary conditions (2): the Monte Carlo method [10]; the approximations of spherical harmonics [11], radiation elements [12], and characteristics [9, 13]; zonal methods [8], and others. A recent trend in the methodology of solving the radiative-transfer equation is the combination of the discrete-ordinate method [7] with the finite-difference method [14, 15] or the finite-element method [3, 16]. The popularity of this approach to the solution of the radiative-transfer equation is explained by the fact that the computational algorithm used in it is relatively simple and compatible with the computational schemes used in the case of different mechanisms of radiative transfer. There are also a number of other methods of attack of this problem; however, a sufficiently reliable and efficient method of solving the radiative-transfer equation is absent at the moment. Each of the existing methods has disadvantages that limit the range of its application. For example, the method of finite elements or finite volumes can be used for solving a limited range of ordinary differential equations (of the first order, the hyperbolic type) for nonuniform high-temperature heat flows; otherwise, physically incorrect results could be obtained (e.g., negative values of the radiation intensity).

To demonstrate problems that could arise in the process of numerical solution of the radiative-transfer equation, we write the radiative-transfer equation in the one-dimensional formulation with account for the isotropic scattering. In this case, Eq. (1) is conveniently written in the form

$$\mu\frac{\partial I}{\partial x} + \alpha I(x,\mu) = Y . \tag{5}$$

Let us solve Eq. (5) in the finite-difference approximation by the explicit and implicit finite-difference schemes for the segment $[1 \to 2]$ of length Δl (the arrow designates the radiation-propagation direction) with homogeneous optico-physical properties:

$$I_2^\mathrm{an} = I_1 \exp[-\tau_\mathrm{at}] + (1 - \exp[-\tau_\mathrm{at}])\,\frac{Y}{\alpha} , \tag{6}$$

$$I_2^\mathrm{im} = I_1 \frac{2-\tau_\mathrm{at}}{2+\tau_\mathrm{at}} + \frac{2Y\Delta l}{\mu(2+\tau_\mathrm{at})} , \tag{7}$$

$$I_2^\mathrm{ex} = I_1 \frac{1}{1+\tau_\mathrm{at}} + \frac{Y\Delta l}{\mu(1+\tau_\mathrm{at})} . \tag{8}$$

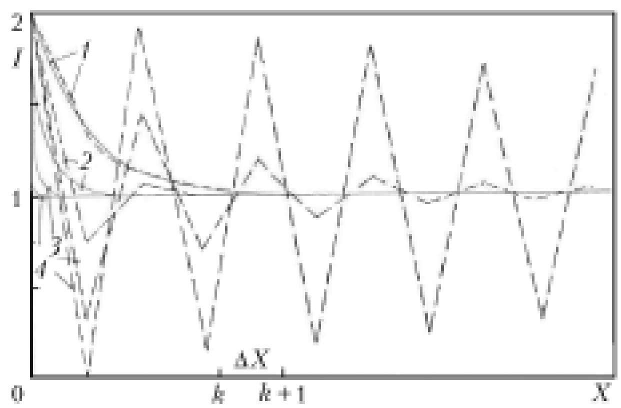

Fig. 1. Illustration of the problem on stable numerical solution of the radiative-transfer equation: μ = 1 (1), 0.3 (2), 0.1 (3), and 0.01 (4) (the solid line is an exact solution; the dashed line is a numerical solution). S = 10, $\alpha\Delta l$ = 1, I_1 = 2.

TABLE 1. Comparison of the Accuracy of the Solutions Obtained by the Explicit and Implicit Difference Schemes (τ_{at} = 1)

γ	$\varepsilon_{ex} = 1 - I_2^{ex}/I_2^{an}$	$\varepsilon_{im} = 1 - I_2^{im}/I_2^{an}$
0.1	0.072	–0.276
0.5	0.025	–0.097
1.0	0.000	0.000
10	–0.046	0.178

The radiation intensity at the boundary of the computational region is I_1.

In the subsequent discussion, we will use the notion of optical thickness of a layer with respect to the amplification: $\gamma = \beta\Delta l\mu$. In this case, Eqs. (6)–(8) can be represented in the form

$$I_2^{an} = I_1 \left(\exp\left[-\tau_{at}\right] + \left(1 - \exp\left[-\tau_{at}\right]\right) \frac{\gamma}{\tau} \right), \tag{9}$$

$$I_2^{im} = I_1 \frac{2(1+\gamma) - \tau_{at}}{2 + \tau_{at}}, \tag{10}$$

$$I_2^{ex} = I_1 \frac{1+\gamma}{1+\tau_{at}}. \tag{11}$$

Analysis of Eqs. (7) and (10) shows that, at a certain relation between the parameters τ_{at} and γ, the radiation intensity can be decreasing, oscillating, or even negative (Fig. 1). In the last-mentioned case, Prof. Fiveland (USA) proposed to reduce the physically incorrect values of the radiation intensity to zero [17]. By way of example, we will consider the distribution of the intensity of the radiation propagating in a plane layer in different directions, calculated analytically and with the use of an implicit numerical scheme (Fig. 1). The implicit scheme gives decreasing, oscillating values of the radiation intensity. Such instabilities do not arise when an explicit difference scheme is used; however, the accuracy of the solution is much lower in this case as compared to that given by the implicit scheme (Table 1). Analogous results are obtained when the radiative-transfer equation is solved in the finite-element approximation.

Thus, to obviate the above-indicated errors, it is necessary to numerically solve the radiative-transfer equation with the use of a computational grid having a fairly small pitch; however, this decreases the rate of calculation, especially in the case of optically thick media (e.g., a furnace medium investigated in the absorption bands of molecular gases). Because of this, a pressing problem is the development of algorithms and methods for solving the radiative-transfer equation that would allow one to exactly estimate the radiant fluxes in inhomogeneous three-dimensional media of complex geometry, in particular in technological processes occurring in a wide temperature range.

Numerical Method of Solving the Radiative-Transfer Equation. We propose a new approach to calculation of the characteristics of radiative heat exchange in the most general and complex case of an absorbing, emitting, and scattering medium having a complex three-dimensional geometry. In accordance with the method proposed, the radiative-transfer equation (1) with boundary conditions (2) is numerically solved with the use of its piecewise-analytic solutions. The results of solution of a number of practical problems by this method [4, 5, 18, 19] show that the method considered is free of many of the disadvantages characteristic of other methods and has a number of advantages that allow it to be used for solving a substantially wider range of problems associated with radiative energy transfer. The method developed by us provides a higher accuracy and rate of solution of the problem studied, as compared to other methods, and, at the same time, calls for smaller computational resources for its realization.

Our method is based on the combination of the discrete-ordinate method [3, 13–16] and the method of ray tracing [13, 19]. In the latter method, the intensity distribution along the trajectory of a radiation ray propagating in a medium is determined with allowance for the optical and geometric properties of this medium and its boundary surface by piecewise-analytic solutions of Eq. (1). The spatial discretization of the computational region is carried out by the finite-element method [16, 20], which allows one to describe complex configurations and retain the compatibility with the computational schemes used for different mechanisms of energy transfer. As a result of the discretization, we obtained a number of discrete elements (N_e) and points (N_p) in which the radiation intensity was calculated.

Then we separated, by the discrete-ordinate method, $N_d = 2 + N_\varphi N_\theta$ radiation-propagation directions. For each separated direction ($k = 1 \dots N_d$), Eq. (1) can be written in the form

$$\frac{\partial}{\partial \mathbf{l}_k} I(\mathbf{r}, \mathbf{l}_k) + a(\mathbf{r})\, I(\mathbf{r}, \mathbf{l}_k) = Y^k, \quad Y^k = \chi B_\lambda + \frac{\sigma}{4\pi} S^k(\mathbf{r}), \tag{12}$$

where $\mathbf{l}_k = \sin\theta_k \cos\varphi_k \mathbf{i} + \sin\theta_k \sin\varphi_k \mathbf{j} + \cos\theta_k \mathbf{k}$; S^k is the integral term in Eq. (1) which, just as the density of the incident radiant flux, is approximated by the Gauss quadrature formula [21] at each point of the computational region ($i = 1 \dots N_p$):

$$Q^i_{\mathrm{f.w}} = \int_{2\pi} I_\lambda(\mathrm{P}_i, \mathbf{l})(\mathbf{l}\cdot\mathbf{n})\, d\Omega \approx \sum_{m=1}^{N_d} A_m I_i^m \vartheta(\mathbf{l}_m, \mathbf{n}_i); \; S_i^k = \int_{4\pi} \rho_\lambda(\mathbf{r}_i, \mathbf{l}^k, \mathbf{l})\, I_\lambda(\mathbf{r}_i, \mathbf{l})\, d\Omega \approx \sum_{m=1}^{N_d} A_m I_i^m p(\mathbf{r}_i, \mathbf{l}_k, \mathbf{l}_m); \tag{13}$$

A_m are the weights of the Gauss quadrature formula. The function $\vartheta(\mathbf{l}_m, \mathbf{n}_i)$ is determined as

$$\vartheta(\mathbf{l}, \mathbf{n}) = \begin{cases} \mathbf{l}\cdot\mathbf{n}, & \mathbf{l}\cdot\mathbf{n} \ge 0; \\ 0, & \mathbf{l}\cdot\mathbf{n} < 0. \end{cases} \tag{14}$$

The scattering indicatrix $p(\mathbf{r}_i, \mathbf{l}_k, \mathbf{l}'_m)$ is usually expressed in terms of the Legendre polynomial [9, 13]:

$$p(\mathbf{r}, \mathbf{l}, \mathbf{l}') = \sum_{n=0}^{N} (2n+1)\, a_n \mathrm{P}(\mathbf{r}, \mathbf{l}, \mathbf{l}'). \tag{15}$$

Note that it can be used, for many urgent problems of radiative transfer, in the approximation [9]

$$p(\mathbf{r}, \mathbf{l}, \mathbf{l}') = a(\mathbf{r})\,[1 - 4\pi\delta(\mathbf{l} - \mathbf{l}')]. \tag{16}$$

Note that the scattering indicatrix of an elementary volume can be calculated using the Mie theory.

After the computational region is divided into finite elements and the radiation-propagation directions are selected, the problem of ray tracing is considered. The scheme of ray tracing is presented in Fig. 2. The radiation intensity at the point P_i is determined by the contribution of the radiation intensities at all the points of the volume in the

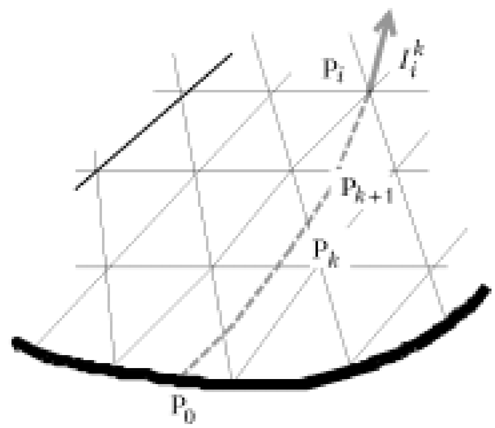

Fig. 2. Scheme of ray tracing.

path of the ray from the boundary point P_0, at which it is known, to the point considered. The desired radiation intensity can be formally represented, with the use of (12), in the form [9]

$$I(P_i) = I(P_0)\,T(P_0, P_i) + \int_{P_0}^{P_i} Y(s)\,T(s, P_i)\,ds\,, \tag{17}$$

where $T(A, B) = \exp\left(-\int_A^B \alpha(s)ds\right)$ is the transmission coefficient along the trajectory of the ray travelling between the arbitrary points A and B. As is seen from Fig. 2, the ray trace intersects discrete elements. In this case, the solution of (10) is a superposition of the solutions for the trace segments in these element. The formulas for calculating the radiation intensity (17) in discrete elements are constructed with allowance for the features of the concrete problem considered. The accuracy of the interpolation of the attenuation coefficient $\alpha(s)$ depends on the degree of inhomogeneity of the medium, and the source function $Y(s)$ is selected depending on the medium inhomogeneity. For example, in the case of a linear interpolation, the expression for $I(P_{k+1})$ has the form

$$I(P_{k+1}) = \begin{cases} \left(I(P_k) - \dfrac{Y_k}{\tilde{\alpha}} + \dfrac{Y_{k+1} - Y_k}{\tilde{\alpha}^2 \Delta_k}\right) \exp[-\tilde{\alpha}\Delta_k] + \dfrac{Y_{k+1}}{\tilde{\alpha}} - \dfrac{Y_{k+1} - Y_k}{\tilde{\alpha}^2 \Delta_k}\,, & \tilde{\alpha}\Delta_k \geq 10^{-5}\,; \\ I(P_k) + \dfrac{\Delta_k}{2}(Y_{k+1} + Y_k)\,, & \tilde{\alpha}\Delta_k < 10^{-5}\,, \end{cases} \tag{18}$$

where $\Delta_k = |P_k, P_{k+1}|$ is the length of the trace segment in a discrete element and $\tilde{\alpha} = (\alpha_{k+1} + \alpha_k)/2$. Thus, the procedure for determining the radiation intensity at the point P_i with the use of (17) and (18) is as follows:

1. The trace of the ray traveling from the point P_i to the point P_0 at the boundary is determined with allowance for the direction of propagation of the ray and its refraction in the bulk of the medium. In this case, the auxiliary points P_k, representing the points of intersection of the ray trace with the faces of the computational grid elements, are determined. It should be noted that, with the method proposed, one can take into account the changes in the refractive index of the medium. This important advantage of the method allows it to be used for calculating the intensity of radiation propagating in very inhomogeneous media.

2. The radiation intensity at the boundary point P_0 is calculated with allowance for the boundary conditions (2).

3. The radiation intensity along the ray trace, including the point P_i, is calculated by the recurrence formula (18).

TABLE 2. Time of Calculation of the Problem on Radiative Transfer for Regions of Different Dimension at a Different Number of Spatial and Angular Discretizations of the Computational Region

	1-*D* problem (N_d = 9)			2-*D* problem (N_d = 17)			3-*D* problem (N_d = 26)		
N	N_p	N_e	t, sec	N_p	N_e	t, sec	N_p	N_e	t, sec
5	5	4	0.0	25	32	0.08	125	384	1.45
7	7	6	0.005	49	72	0.155	343	1286	4.45
9	9	8	0.0075	81	128	0.26	729	3072	10.38
11	11	10	0.009	121	200	0.39	1331	6000	20.87
15	15	14	0.012	225	392	0.785	3375	17640	70.65
17	17	16	0.015	289	512	1.054	4913	26112	110.65

Since the right side of the radiative-transfer equation (1) and the boundary conditions (2) are dependent on the desired values of the radiation intensity, it is necessary to perform iteration to refine the integral terms in expressions (1) and (2). The computational algorithm used for determining the radiative-transfer characteristics includes the following operations:

1. The initial values of the radiation intensity are prescribed. In particular, it may be assumed that $I_i^k = 0$ for all the grid points and radiation-propagation directions.

2. The density of the radiation flux incident on the boundary of the medium is determined using formulas (12) and (13).

3. The integral term in the radiative-transfer equation $S^k(\mathbf{r})$ is calculated by formula (4) for each of the radiation propagation directions $k = 1 \ldots N_d$ selected. Then the radiation intensity I_i^k at each point of the computational region ($i = 1 \ldots N_p$) is calculated by the above-described scheme of ray tracing with the use of the piecewise-analytic solutions (17) and (18). The relative error in the values obtained in the neighboring iterations is calculated by the formula

$$\delta = \max_{i,k} \left| 1 - I_i^{k,s} / I_i^{k,s+1} \right| . \tag{19}$$

4. If the desired accuracy is not attained ($\delta > \delta_0$, where δ_0 is the accuracy prescribed in advance), the calculation is repeated, beginning with Sec. 2.

5. If conditions (19) are fulfilled, we obtain a basic set of radiation intensities I_i^k for determining the integral terms in Eq. (1) at boundary conditions (2). Then, using the above-described scheme of ray tracing, one may calculate the radiation intensity at any point of the computational region and in any radiation propagation direction without resorting to the interpolation between the basic points and basic directions.

Main Advantages of the Method Proposed for Solving the Radiative-Transfer Equation. The above-described computational algorithm allows one to calculate the characteristics of radiative transfer in absorbing, emitting, scattering, and reflecting media of complex geometry. In Fig. 3, the possibilities of the method proposed are demonstrated by the example of the results of calculation of the density of a radiant flux incident on the boundary surface of four absorbing and emitting regions having a complex three-dimensional geometry. It is seen from the results presented in this figure that the incident radiant flux is most inhomogeneous in the neighborhood of the regions with a sharply varying surface curvature.

The algorithm proposed works well in the case of media with small and large optical densities, for which other methods can give physically incorrect results. This algorithm cannot give, in principle, negative or oscillating values of the radiation intensity.

Since items 3 and 4 of the computational algorithm represent separate problems, the intensity of radiation can be calculated for each prescribed direction of its propagation with the use of multiprocessor computational systems.

When our results are compared with the results of other authors and known analytical solutions, it is apparent that the method proposed provides a high accuracy of calculations. For example, our results agree with the known analytical solutions within the computer error.

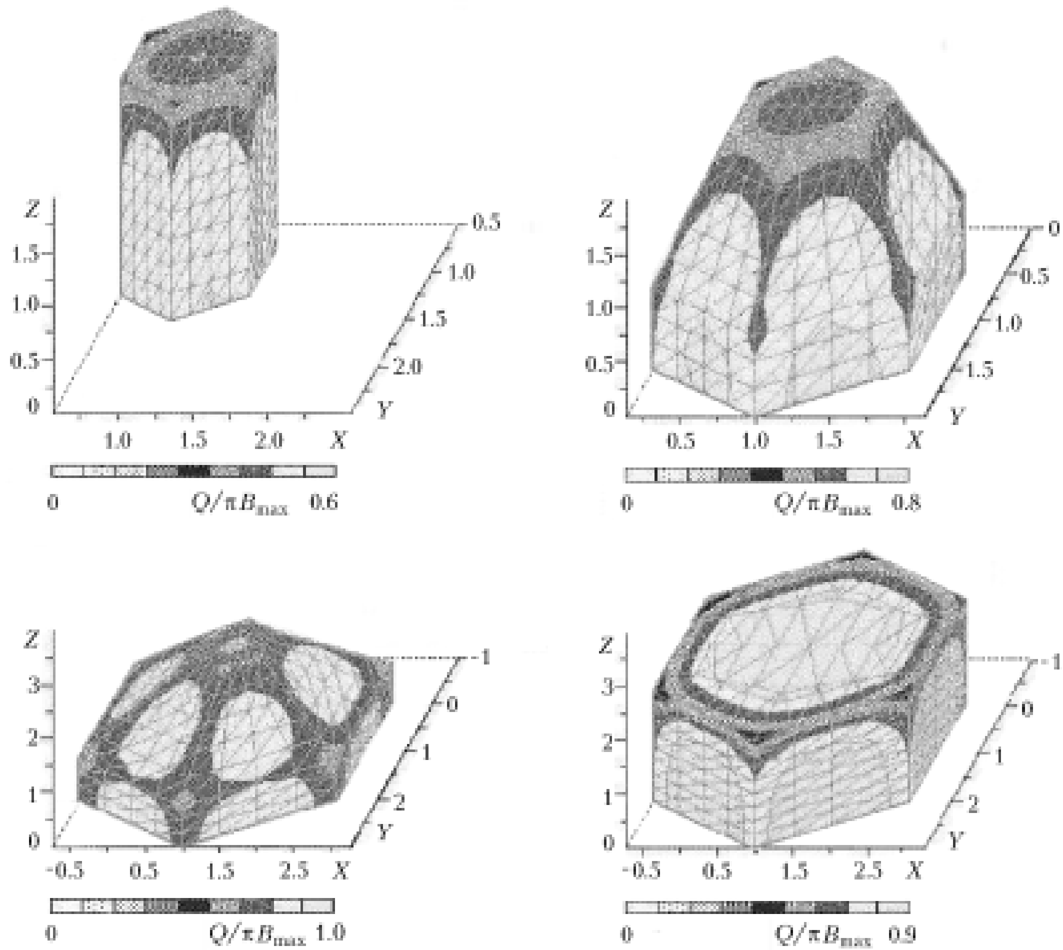

Fig. 3. Distribution of the density of the radiant flux incident on the boundary surface of an absorbing and emitting medium (χ = 1.0 m^{-1}, the boundaries are transparent, the lower boundary is mirror).

One of the most important advantages of the method proposed, as compared to the finite-difference and finite-element methods [16, 17], is that it allows one to obtain a solution in explicit form without resorting to solving the system of algebraic differential equations. This makes it possible to substantially decrease the time of calculations and the amount of random access memory. Table 2 presents, as a case in point, the times of radiative transfer in media of simple 1–3-*D* geometries (segment, square, cube), calculated with the use of different numbers of spatial and angular discretizations on a low-power computer with a 133-MHz processor. These data were obtained for media with an optical density equal to unity with the use of three iterations. For media with optical densities from 0 to 100, the time of calculation differed from the time given in the table by no more than 5%. The results obtained show that, in the three-dimensional case, an element-direction iteration is calculated for $\sim 5 \cdot 10^{-5}$ sec with the use of the algorithm proposed and the above-mentioned computer. The solution of this problem by the finite-element and discrete-ordinate methods [16] using the same computer takes almost 100 times longer ($4.5 \cdot 10^{-3}$ sec for an element-direction iteration). For comparison, we also give the data obtained in [16], where the calculation of a point-direction iteration by the discrete-ordinate and finite-difference methods took $2.2 \cdot 10^{-4}$ sec of operation of a VAX 11/785 processor. As is seen from the comparison presented, the algorithm proposed provides a high rate of calculations due to the absence of the need for solving the system of algebraic equations and because of the use of a special algorithm for determining the ray trace in a region covered with an irregular grid. With this algorithm, neighboring elements having common faces are determined and stored in the process of construction of a computational finite-element grid. After this grid is constructed, the determination of the ray trace becomes algorithmically trivial.

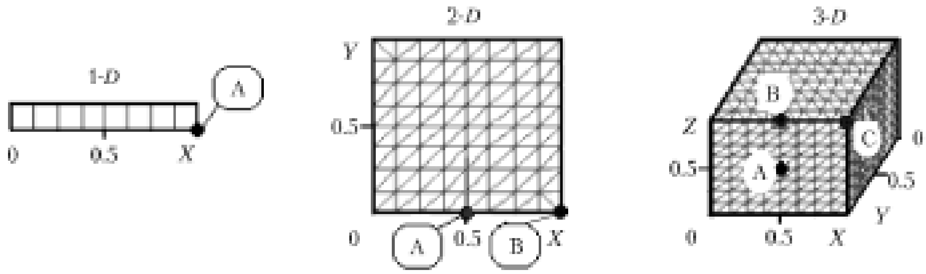

Fig. 4. Computational schemes for one- (1-*D*), two- (2-*D*), and three-dimensional (3-*D*) problems. A, B, and C are the points of the system, used for comparison of radiant fluxes.

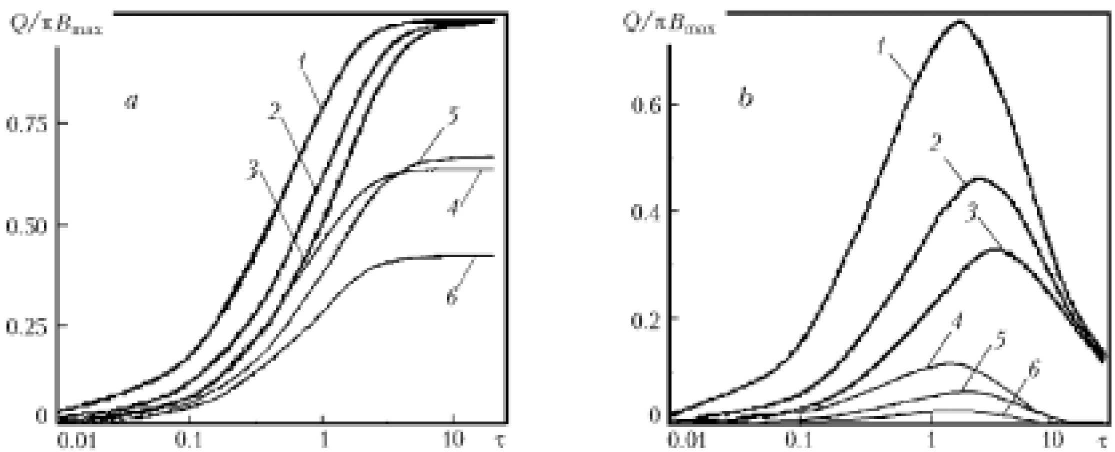

Fig. 5. Dependence of the reduced density of the incident radiant flux on the optical density with respect to the absorption, determined for homogeneous (a) and inhomogeneous (b) media: 1) one-dimensional problem; 2 and 4) two-dimensional problem, 3, 5, and 6) three-dimensional problem [1, 2, and 3) radiant flux at the point A; 4 and 5) radiant flux at the point B; 6) radiant flux at the point C (the arrangement of points A, B, and C is shown in Fig. 4)].

It should be noted that the method proposed is similar, in some sense, to the Monte Carlo method of direct physical simulation. However, in our method, unlike the Monte Carlo method, the statistical approach is used instead of the deterministic approach for determining the desired quantities. At the same time, it allows one to solve all of the problems that are solved by the Monte Carlo method and substantially decreases the calculation time, especially in the case of optically thick media.

Examples of Calculation of Radiative-Transfer Characteristics for One-, Two-, and Three-Dimensional Problems. A traditional procedure used in solving certain physical and technical problems is decreasing the dimension of the problem. In some cases, this can be done at the expense of the symmetry of the problem; however, in the majority of cases, the dimension of a problem is decreased because of substantial computational difficulties. With the examples considered in this section, we will show that, in the case of calculation of the radiative heat transfer in media with absorption and scattering, a decrease in the dimension of the problem can lead to large errors. The above-described method of solving the radiative-transfer equation will be used for calculations. Figure 4 presents computational grids for one-, two-, and three-dimensional problems. The equivalent points of these grids, used for comparison of incident radiant fluxes, are marked by identical letters. We considered homogeneous and inhomogeneous media. It was assumed that a homogeneous medium is a gas medium with a constant temperature and an inhomogeneous medium is a gaseous region with a hot core at its center. An example of such a medium is the space of a furnace with a flame inside it. It was assumed that the Schlichting temperature distribution is realized inside the inhomogeneous region. In this case, the radiation intensity at the colder boundaries of the region was ten times lower than the radiation intensity

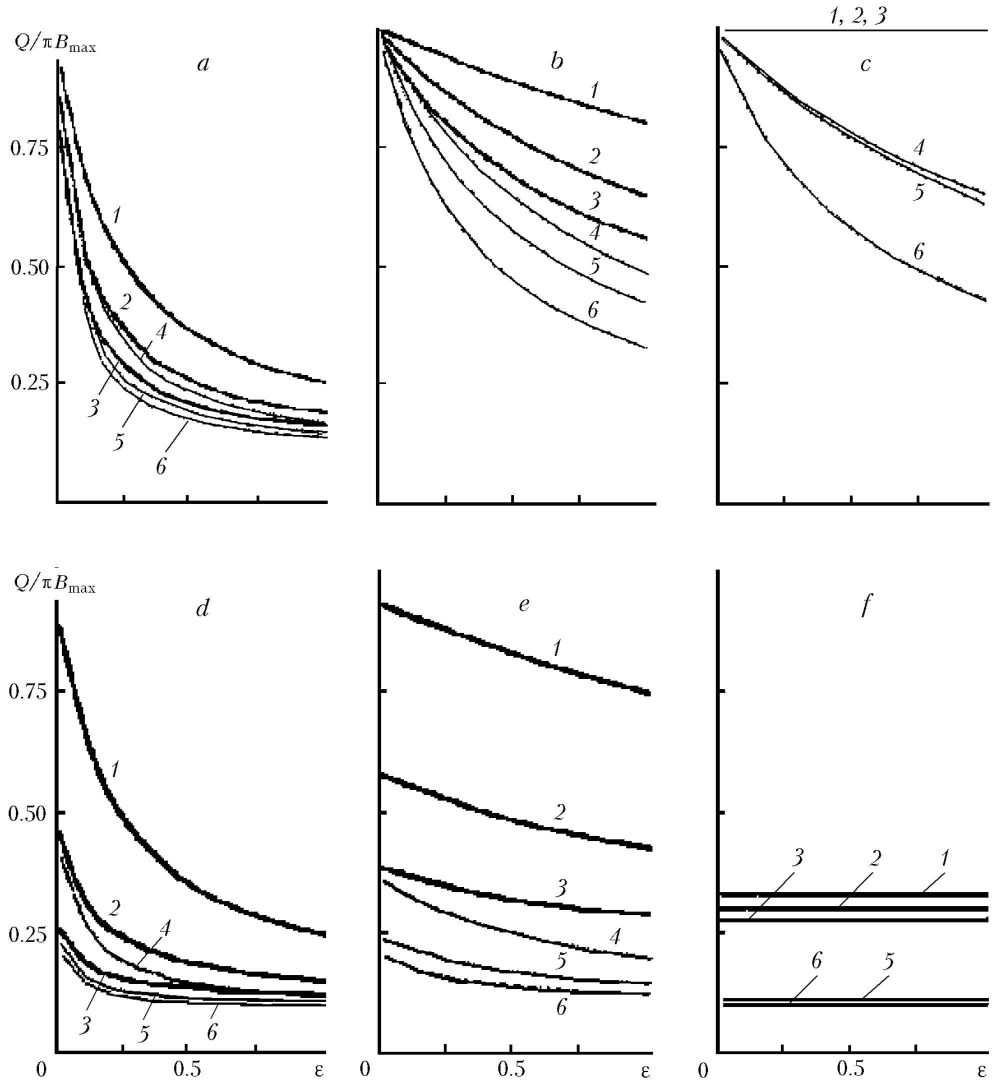

Fig. 6. Dependence of the reduced density of the incident radiant flux on the blackness of the boundary surface, determined for homogeneous (a, b, c) and inhomogeneous (d, e, f) media: τ = 0.1 (a, d), 1.0 (b, e), and 10 (c, f). Designations 1–6 are identical to those in Fig. 5.

at its center. The maximum temperature at the center of the inhomogeneous region was equal to the temperature of the homogeneous region. In the calculations, this maximum temperature was 2000 K and the temperature at the boundaries was 1120 K.

Figure 5 presents the results of parametric investigations of the dependence of the reduced density of the incident radiant flux Q on the optical density τ with respect to the absorption for homogeneous and inhomogeneous media. In this case, τ is determined from the expression [3]

$$\tau = \frac{1}{U^{\frac{d-1}{d}}} \int \dots \int \chi(\mathbf{r})\, dU . \qquad (20)$$

Here, d is the dimension of the problem (d can be equal to 1, 2, or 3), U is the geometric dimension of the region (thickness, area, or volume, respectively), and the d-dimensional integral is calculated throughout the absorbing region. Analysis of the results obtained shows that, both in the case of a medium with a low optical density ($\tau \sim 0.1$) and in the case of a medium with a high optical density ($\tau \sim 10$), a decrease in the dimension of the problem does not lead to large errors in the calculation of the incident radiant flux. However, at intermediate values of the optical density of a medium ($\tau \sim 1$), this error increases substantially and can reach 100%. This points to the fact that the dimension of a problem on radiative transfer can be decreased only after the statement of this problem is thoroughly analyzed. In this case, prominence should be given to the optical density of the medium studied.

Figure 6 presents results of parametric investigation of the dependence of the reduced density of an incident radiant flux Q on the blackness ε of the boundary surface of homogeneous and inhomogeneous media having different optical densities. In this case, the error introduced by the decrease in the dimension of the problem is most significant for media with a small optical density, and this error decreases with increase in the optical density.

CONCLUSIONS

1. An efficient numerical method of solving the radiative-transfer equation for absorbing, emitting, and scattering media of complex geometry has been proposed.

2. The method proposed has been compared with known analogous methods.

3. The errors arising as a result of a decrease in the dimension of the problem on radiative transfer in an absorbing, emitting, and scattering medium have been calculated and analyzed using the algorithm developed by us.

4. The algorithm developed can be used for investigating the radiative transfer in energy-plant furnaces, nuclear reactors, and high-temperature chemical reactors as well as in the atmosphere and the cosmos.

NOTATION

$a(\mathbf{r})$, doubled fraction of the radiation scattered backward by an elementary volume of the medium; a_n, coefficients of the Legendre polynomial; $B_\lambda(T)$, spectral intensity of the blackbody radiation at a temperature T; $B_{\max}(T)$, maximum intensity of the blackbody radiation at a temperature T; I, radiation intensity; $I_\lambda(\mathbf{r}, \mathbf{l})$, spectral intensity of the radiation at the point $\mathbf{r}$ in the direction $\mathbf{l}$; $I_{0\lambda}(\mathrm{P}, \mathbf{l})$, spectral intensity of the intrinsic or outer radiation at the boundary point P; $\mathbf{l}$ and $\mathbf{l}'$, directions of the ray propagation; $Y(s)$, source function; $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, unit vectors of coordinate axes; N, number of space points along each of the axes; Δl, geometric length of the path of travel of the optical radiation; N_e, number of discrete elements; N_p, number of points; N_d, number of radiation-propagation directions; n, external normal to the boundary; N_φ and N_θ, number of directions in the horizontal ($0 \le \varphi \le 2\pi$) and vertical ($0 \le \theta \le \pi$) planes, respectively; $p_\lambda(\mathbf{r}, \mathbf{l}, \mathbf{l}')$, indicatrix of scattering of radiation in the process of its interaction with an elementary volume of the medium; $q_\mathrm{w}^\mathrm{r}(\mathrm{P})$, local density of the resulting radiation flux incident on a heat-absorbing surface at the point P; $\mathbf{Q}_\mathrm{r}$, volume density of radiation sources/heat flows; $Q_{\mathrm{f.w}}(\mathbf{r})$, density of the radiant flux incident on the boundary of the medium; $\mathbf{r}$, radius-vector; S, integral term of Eq. (1); s, path of travel of the ray; T, temperature; t, time of numerical calculation of the problem; X, Y, Z, coordinate axes; x, coordinate; $\alpha = \chi + \sigma$, coefficient of the complete attenuation of radiation by the medium; $\chi_\lambda(\mathbf{r})$ and $\sigma_\lambda(\mathbf{r})$, spectral coefficients of absorption and scattering, respectively; $\beta = Y/I_1$, amplification coefficient of the medium; δ, error; ε, degree of blackness; θ, angle between the radiation-propagation direction and the external perpendicular to the boundary surface of the layer; $\mu = \cos\theta$; λ, is the electromagnetic radiation wavelength; $\tau_\mathrm{at} = \alpha\Delta l/\mu$, optical thickness of the layer with respect to the attenuation of radiation along the ray propagation direction; τ, optical density of the medium with respect to the absorption; θ_k and φ_k, polar and azimuth angles in the spherical coordinate system; $\Delta_k = \left|\mathrm{P}_k, \mathrm{P}_{k+1}\right|$, length of the trace inside a discrete element; $\rho_\lambda(\mathrm{P}, \mathbf{l}, \mathbf{l}')$, spectral coefficient of reflection from the boundary; Ω, solid angle. Subscripts: an, analytical solution; im, implicit solution; ex, explicit solution; p, point; d, direction; 0, points at the boundary; r, radiative; w, surface; f.w, incident on the surface; λ, spectral characteristics; at, attenuation; max, maximum; e, element.

REFERENCES

1. T. L. Bergman and R. Viskanta, Radiation heat transfer in manufacturing and material processing, in: *Proc. First Int. Symp. on Radiation Transfer*, Kasadasi, Turkey (1995), pp. 13–39.
2. R. L. Anderson, Review of temperature measurements in the semiconductor industry, *SPIE*, **1392**, 437–451 (1990).

3. M. L. German, *Influence of the Optical Properties of a Two-Phase Medium and Its Boundary Surface on the Radiative Heat Transfer in Furnace Chambers*, Candidate's Dissertation (in Physics and Mathematics), Minsk (1993).
4. M. L. German, V. A. Borodulya, and E. F. Nogotov, Thermal calculation of the furnace chamber of a fire-tube boiler with a dead-end furnace, *Inzh.-Fiz. Zh.*, **73**, No. 6, 1191–1201 (2000).
5. M. L. German and P. S. Grinchuk, Mathematical model for calculating the heat-protection properties of the composite material "ceramic microspheres–binder," *Inzh.-Fiz. Zh.*, **75**, No. 6, 43–53 (2002).
6. M. L. German, V. P. Nekrasov, E. F. Nogotov, and N. I. Lemesh, Some aspects of energy conservation with the use of double-glass sheets with a different light transmission, in: *Proc. Sci.-Eng. Conf. "Energy-Effective Systems of Lighting in Residential, Public, and Industrial Buildings"* [in Russian], Minsk (1999), pp. 12–14.
7. S. Chandrasekhar, *Radiative Transfer*, Dover, New York (1960).
8. H. C. Hottel and A. F. Sarofim, *Radiative Transfer*, McGraw-Hill, New York (1967).
9. K. S. Adzerikho, E. F. Nogotov, and V. P. Trofimov, *Radiative Heat Transfer in Two-Phase Media* [in Russian], Nauka i Tekhnika, Minsk (1987).
10. R. Siegel and J. R. Howell, in: *Thermal Radiation Heat Transfer*, 3rd ed., Hemisphere, Washington DC (1992), pp. 795–804.
11. M. Menguc and R. Viskanta, Radiative transfer in three-dimensional rectangular enclosures, *JQSRT*, **35**, 533–549 (1985).
12. S. Maruyama and T. Aihara, Radiative heat transfer of arbitrary 3-D participating media and surfaces with non-participating media by a generalized numerical method REM, in: *Proc. First Int. Symp. on Radiation Transfer*, Kasadasi, Turkey (1995), pp. 153–167.
13. M. Menguc and S. Manickavasagam, Inverse radiation problem in axisymmetric cylindrical media, *J. Thermophys. Heat Transfer*, **23**, 479–489 (1993).
14. W. A. Fiveland, Discrete-ordinate solutions of the radiative transport equation for rectangular enclosures, *J. Heat Transfer*, **106**, 699–706 (1984).
15. J. S. Truelove, Three-dimensional radiation in absorbing-emitting-scattering media using the discrete-ordinates approximation, *JQSRT*, **39**, No. 1, 27–31 (1988).
16. M. L. German, V. P. Nekrasov, and E. F. Nogotov, Numerical method for calculating the radiation transfer in two-phase media of complex geometry, *Dokl. Akad. Nauk Belarusi*, **40**, No. 3, 122–126 (1996).
17. W. A. Fiveland, Solution of the three-dimensional equation of radiative heat transfer by the discrete-ordinate method, *Aérokosm. Tekh.*, No. 9, 79–88 (1989).
18. M. L. German, P. M. Kolesnikov, and E. F. Nogotov, An effective numerical method for calculation of neutron transfer, in: *Proc. Int. Sci. Conf. "Thermophysics-98"* [in Russian], Obninsk (1998), pp. 301–308.
19. M. L. German, V. P. Nekrasov, E. F. Nogotov, and Yu. V. Belyaev, Influence of the optical properties of decorative materials on the illuminance of a room, in: *Proc. Sci.-Eng. Conf. "Nontraditional Energy-Efficient Systems of Lighting in Residential, Public, and Industrial Buildings"* [in Russian], NIPTIS, Minsk (1997), pp. 23–26.
20. O. C. Zienkiewicz, *The Finite Element Method in Engineering Sciences*, McGraw-Hill, London (1971).
21. A. N. Lowan, N. Davids, and A. Levenson, Table of the zeros of the Legendre polynomials of order 1–16 and the weight coefficients for Gauss mechanical quadrature formula, *Bull. Amer. Math. Soc.*, **48**, 739–742 (1942).